

#### Introduction

This document provides a high-level mathematical formulation and explanation of the clearing algorithm used to conduct the Reliability Pricing Model (RPM) Base Residual Auction (BRA). The intent of this document is to provide a better understanding of the mechanism that clears the Base Residual Auction and also to address commonly asked questions.

Some of the logic employed in the optimization such as Flexible Self-Scheduling, Tie-Breaking, and Make-Whole, is not discussed in this document as the intent is to explain the core formulation. The formulation for clearing Incremental Auctions is also not included.

### **Model Overview**

The model for the BRA is setup using the Planning Parameters posted on pjm.com. The applicable Capacity Emergency Transfer Limits (CETL) and VRR curves are applied to each constrained LDA and the unconstrained region. Dependant upon the level of nesting, each LDA is assigned a parent LDA such that all constrained LDAs are either directly, or indirectly, children of the unconstrained region. The original concept of "rest of Market" is not employed in RPMO. Resources are represented in the lowest level LDA modeled in that BRA that they physically reside in. Transmission offers are modeled as price sensitive increments to the CETL of the sink LDA specified in the offer.

## Application of the Variable Resource Requirement (VRR) Curve

A VRR curve is defined for each constrained LDA and the unconstrained region for a BRA. It represents the relationship between cleared capacity and the value in \$/MW-day for that level of capacity. Therefore, it is critical that the optimization produces cleared MW and price quantities for each constrained LDA and the unconstrained region that are points on their respective VRR curves.

The application of this curve in the unconstrained region follows the traditional MIP model where the shadow price is derived from satisfying the supply and demand constraints. More complex is the application of the VRR curve in a constrained LDA. Since the constrained LDA is a sub-region of the unconstrained region, the traditional MIP model derives the shadow price of the constrained LDA as the sum of satisfying the supply and demand constraints for the constrained LDA, and that of the unconstrained region, as resource within the constrained LDA relieves both demand constraints. This solution however does not comply with requirement that the solution be a point on the applicable VRR curve. In this case, the constrained LDA shadow price is set according to the defined VRR curve.



### **Objective Function**

The RPM Optimization (RPMO) seeks to minimize the cost of procuring unforced capacity (UCAP) given the variable resource requirements and Planning Parameters modeled in the BRA. The objective function, shown in the following section, is solved using a mixed integer programming (MIP) method where the integer variables are the commitment statuses of resource and qualifying transmission upgrade offer segments.

The objective may be described with the following mathematical terms:

$$\begin{aligned} & \text{M inimize Z} = \sum_{i,seg} \text{BaseOfferPrice}_{seg} \times BaseOfferMWCleared}_{seg} \\ & + \sum_{xm,seg} \text{X missionOfferPrice}_{seg} \times XmissionOfferMWCleared}_{seg} \\ & - \sum_{seg} VRRSeg \text{Price}_{seg} \times VRRSegMWCleared}_{seg} \\ & - \sum_{sr,seg} SrVRRSeg \text{Price}_{sr,seg} \times SrVRRSegMWCleared}_{sr,seg} \end{aligned}$$

where,

BaseOfferPrice<sub>i,seg</sub>: The offer price of segment seg;

 $BaseOfferMWCleared_{i,seg}$ : The cleared UCAP MW amount for segment seg;

XmissionOfferPrice xm, seg: The offer price of transmission upgrade segment seg;

*XmissionOfferMWCleared*<sub>xm,seg</sub>: The cleared UCAP MW amount for transmission upgrade segment seg;

*VRRS*eg*P*rice<sub>seg</sub>: The price from the VRR curve on segment seg for the unconstrained region;

 $\mathit{VRRS}\mathit{egM}\,\mathit{WCleared}_\mathit{seg}$ : The cleared UCAP MW from the VRR curve on segment  $\mathit{seg}$  for the unconstrained region;

SrVRRSegPrice<sub>sr,seg</sub>: The price from the VRR curve on segment seg for the LDA sr; SrVRRSegMWCleared<sub>sr,seg</sub>: The cleared UCAP MW from the VRR curve on segment seg for the LDA sr;

#### **Constraints**

The constraints in this section are categorized based on their application. A description of each constraint variable as well as

This major constraints modeled in the optimization are described below.

Dispatchable Resource Offer Constraints

1)  $MinBaseOfferMW_{seg} \leq BaseOfferMWCleared_{seg} \leq MaxBaseOfferMW_{seg}$ 



where,

 ${\it MinBaseOfferMW}_{\it seg}$ : The minimum UCAP MW offered on resource segment  $\it seg$ ;  ${\it MaxBaseOfferMW}_{\it seg}$ : The maximum UCAP MW offered on resource segment  $\it seg$ ;  ${\it BaseOfferMWCleared}_{\it see}$ : The cleared UCAP MW on resource segment  $\it seg$ .

- 2)  $BaseOfferMWCleared_{seg} \leq BaseCommitted_{seg} \times MaxBaseOfferMW_{seg}$
- 3)  $BaseOfferMWCleared_{seg} \geq BaseCommitted_{seg} \times MinBaseOfferMW_{seg}$  where,

*BaseCommitted*<sub>seg</sub>: {0,1} decision variable indicating the commitment status of the resource segment.

#### Fixed Offer Constraints

4)  $BaseOfferMWCleared_{seg} = MaxBaseOfferMW_{seg}$ 

### Transmission Upgrade Constraints

5)  $MinXmissionOfferMW_{seg} \leq XmissionOfferMWCleared_{seg} \leq MaxXmissionOfferMW_{seg}$  where,

 ${\it MinXmissionOfferMW}_{\it seg}$ : The minimum UCAP MW offered on transmission segment  ${\it seg}$ ;

 $MaxXmissionOfferMW_{seg}$ : The maximum UCAP MW offered on transmission segment seg;

XmissionOfferMWCleared<sub>see</sub>: The cleared UCAP MW on transmission segment seg.

- 6)  $XmissionOfferMWCleared_{seg} \leq XmissionCommitted_{seg} \times MaxXmissionOfferMW_{seg}$
- 7)  $XmissionOfferMWCleared_{seg} \ge XmissionCommitted_{seg} \times MinXmissionOfferMW_{seg}$  where,

 $XmissionCommitted_{seg}$ : {0,1} decision variable indicating the commitment status of the transmission segment.

### Locational Deliverability Area (LDA) Constraints

8)  $0 \le SrVRRSegMWCleared_{sr,seg} \le MaxSrVRRSegMW_{sr,seg}$  where,



 $SrVRRSegMWCleared_{sr,seg}$ : The cleared UCAP MW from the linearized VRR segment seg from LDA sr;

 $MaxSrVRRSegMW_{sr,seg}$ : The maximum UCAP MW from the linearized VRR segment seg from LDA sr.

9) Im  $port_{sr} \leq \text{Im } portCapability_{sr} + \sum_{xm,seg} XmissionOfferMWCleared_{seg}$  where,

 $Im port_{sr}$ : The total import capability of LDA sr;

Im  $portCapability_{sr}$ : The CETL of LDA sr plus any Incremental CTRs sinking in LDA sr,

 $\sum_{sr,seg}$  *XmissionOfferMWCleared*  $_{seg}$ : The sum of all cleared transmission offer segments seg sinking in LDA sr.

10) 
$$SrCapacityObligation = \sum_{seg} SrVRRSegMWCleared_{sr,seg}$$
 where,

*SrCapacityObligation*: The total LDA unforced capacity obligation that must be met with procured capacity,

# Unconstrained Region Constraints

11)  $0 \le VRRSegMWCleared_{seg} \le MaxVRRSegMW_{seg}$  where,

*VRRSegMWCleared* seg: The cleared UCAP MW from the linearized VRR segment seg;

 $\mathit{MaxVRRSegMW}_{\mathit{seg}}$ : The maximum UCAP MW from the linearized VRR segment  $\mathit{seg}$ ;

12) 
$$RTOCapacityObligation = \sum_{seg} VRRSegMWCleared_{seg}$$
 where,

RTOCapacityObligation: The total RTO unforced capacity obligation that must be met with procured capacity